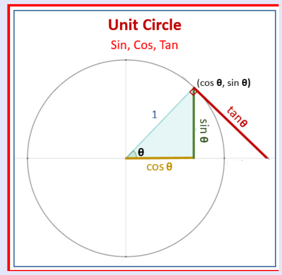
Trigonometry

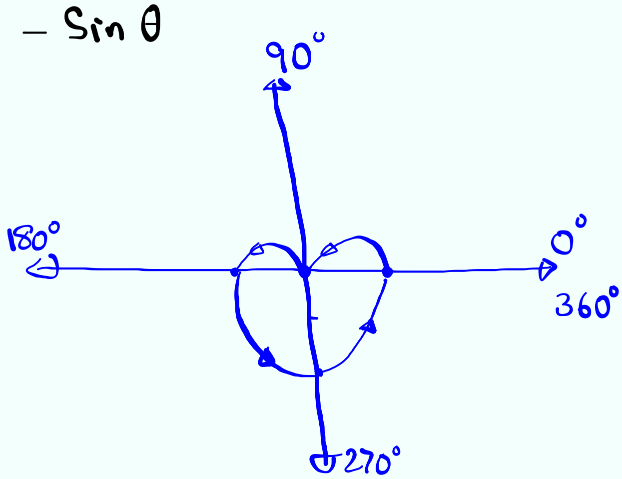
Lecture 52



Feb 19-8:47 AM

Graph $r = 1 - \sin\theta$

θ	r
0°	1
90°	0
180°	1
270°	2
360°	1



Dec 5-10:27 AM

Graph $r = 1 - 2\sin\theta$

θ	r
✓ $0°$	1
$90°$	-1
$180°$	1
$270°$	3
$360°$	1

$r = 0 \rightarrow \theta = 30°$

$r = 0 \rightarrow \theta = 150°$

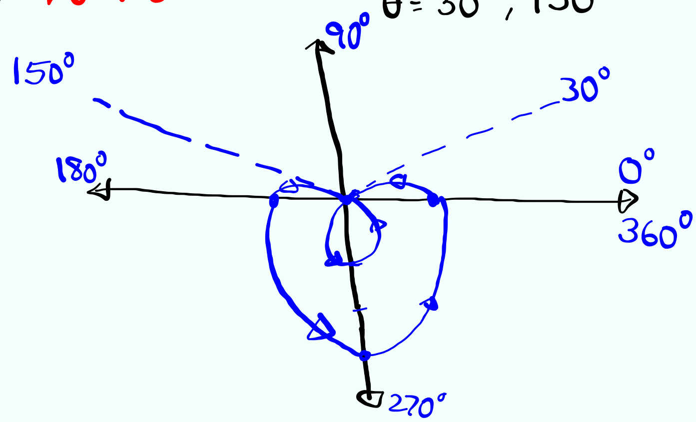
$r = 0$

$1 - 2\sin\theta = 0$

$-2\sin\theta = -1$

$\sin\theta = \frac{1}{2}$

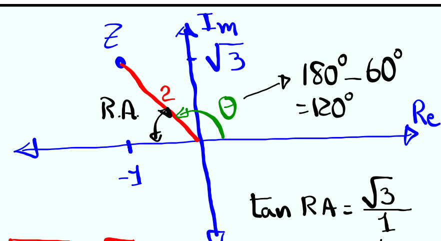
$\theta = 30°,\ 150°$



Dec 5-10:31 AM

$Z = -1 + \sqrt{3}\, i$

1) Plot Z



$\tan RA = \frac{\sqrt{3}}{1}$

$RA = \tan^{-1}\sqrt{3}$

$RA = 60°$

2) $|Z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$r = |Z|$

3) write Z in polar or trig form.

$Z = r(\cos\theta + i\sin\theta) = 2(\cos 120° + i\sin 120°)$

4) Find Z^3

$Z^3 = 2^3(\cos 3\cdot 120° + i\sin 3\cdot 120°)$

$= 8(\cos 360° + i\sin 360°)$

$= 8(1 + i\cdot 0)$

$= \boxed{8}$

Dec 5-10:37 AM

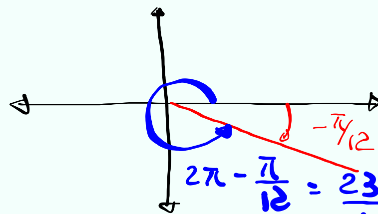
$Z_1 = 2\,\text{Cis}\,\frac{\pi}{4}$ $\qquad Z_2 = 5\,\text{Cis}\,\frac{\pi}{3}$

$Z_1^4 = 2^4\,\text{Cis}\,4\cdot\frac{\pi}{4} = 16\,\text{Cis}\,\pi$

$Z_2^3 = 5^3\,\text{Cis}\,3\cdot\frac{\pi}{3} = 125\,\text{Cis}\,\pi$

$Z_1 Z_2 = 2\cdot 5\,\text{Cis}\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = 10\,\text{Cis}\,\frac{7\pi}{12}$

$\frac{Z_1}{Z_2} = \frac{2}{5}\,\text{Cis}\left(\frac{\pi}{4}-\frac{\pi}{3}\right) = \frac{2}{5}\,\text{Cis}\left(\frac{-\pi}{12}\right) = \frac{2}{5}\,\text{Cis}\,\frac{23\pi}{12}$



$2\pi - \frac{\pi}{12} = \frac{23\pi}{12}$

Dec 5-10:44 AM

Find $\left(\frac{1}{2}+\frac{1}{2}i\right)^{10}$

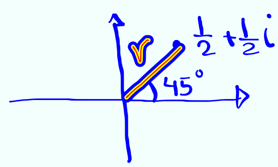
$\frac{1}{2}+\frac{1}{2}i$

$= \frac{\sqrt{2}}{2}\left(\cos 45^\circ + i\sin 45^\circ\right)$

$\frac{1}{2}+\frac{1}{2}i$

Re. $= \frac{1}{2}$

Im. $= \frac{1}{2}$



$|z| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$

$\left[\frac{\sqrt{2}}{2}\left(\cos 45^\circ + i\sin 45^\circ\right)\right]^{10} = \frac{(\sqrt{2})^{10}}{2^{10}}\left(\cos 10\cdot 45^\circ + i\sin 10\cdot 45^\circ\right)$

$= \frac{32}{1024}\left[\cos \underbrace{450^\circ}_{360^\circ+90^\circ} + i\sin 450^\circ\right]$

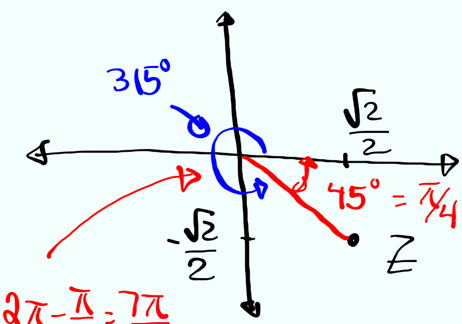
$= \frac{1}{32}\left[\underbrace{\cos 90^\circ}_{0} + i\,\underbrace{\sin 90^\circ}_{1}\right] = \boxed{\frac{1}{32}i}$

Dec 5-10:50 AM

$$\Big(\overbrace{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i}^{z}\Big)^{12}$$

$$|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$$



$$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$\big(1(\cos 315° + i\sin 315°)\big)^{12}$$

$$= 1^{12}\left[\cos \cancel{12}^{\,3}\cdot\frac{7\pi}{\cancel{4}} + i\sin \cancel{12}\cdot\frac{7\pi}{\cancel{4}}\right]$$

$$= \cos 21\pi + i\sin 21\pi$$

$$21\pi = 20\pi + \pi$$

$$= 10\cdot 2\pi + \pi$$

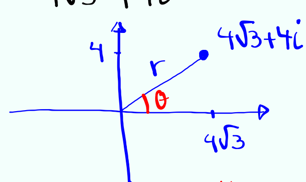
$$= \cancel{\cos}\,\pi \to -1 \quad + i\,\cancel{\sin}\,\pi \to 0$$

$$= \boxed{-1}$$

Dec 5-10:57 AM

Find cube roots of $4\sqrt{3} + 4i$

3 Answers



$$r = |z| = \sqrt{(4\sqrt{3})^2 + 4^2}$$

$$= \sqrt{16\cdot 3 + 16}$$

$$= \sqrt{64} = 8$$

$$\tan\theta = \frac{4}{4\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{\sqrt{3}}{3}$$

$$\theta = 30°$$

$$4\sqrt{3} + 4i = 8(\cos 30° + i\sin 30°)$$

Cube roots

$n = 3$

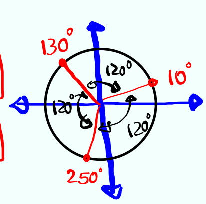
$$\sqrt[3]{8}\left(\cos\frac{30° + k\cdot 360°}{3} + i\sin\frac{30° + k\cdot 360°}{3}\right)$$

$$= 2\left[\cos(10° + k\cdot 120°) + i\sin(10° + k\cdot 120°)\right]$$

$k=0$ $\quad 2[\cos 10° + i\sin 10°]$

$k=1$ $\quad 2[\cos 130° + i\sin 130°]$

$k=2$ $\quad 2[\cos 250° + i\sin 250°]$

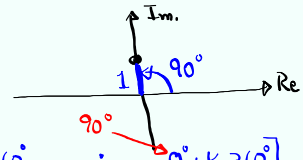


Dec 5-11:05 AM

Find $\underbrace{\text{fifth roots}}_{\text{5 Ans.}}$ of i.

$i = 0 + i$

Re. = 0 Im. 1



$n = 5$

$K = 0, 1, 2, 3, 4$

$$\sqrt[5]{1}\left[\cos\frac{\theta^\circ + K\cdot 360^\circ}{5} + i\sin\frac{\theta^\circ + K\cdot 360^\circ}{5}\right]$$

$$= 1\left[\cos\frac{90^\circ + K\cdot 360^\circ}{5} + i\sin\frac{90^\circ + K\cdot 360^\circ}{5}\right]$$

$$= \cos[18^\circ + K\cdot 72^\circ] + i\sin[18^\circ + K\cdot 72^\circ]$$

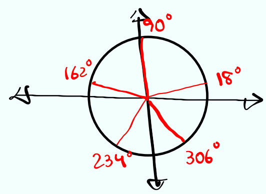
$K=0$ $\text{Cis } 18^\circ$

$K=1$ $\text{Cis } 90^\circ$

$K=2$ $\text{Cis } 162^\circ$

$K=3$ $\text{Cis } 234^\circ$

$K=4$ $\text{Cis } 306^\circ$



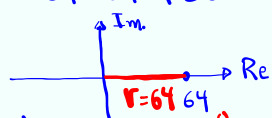
Dec 5-11:13 AM

Find the $\underbrace{\text{sixth roots}}_{\text{6 Ans.}}$ of 64

$64 = 64 + 0i$

$n = 6$

$K = 0, 1, 2, \dots, 5$



$\theta = 0^\circ$

$$64 = 64(\cos 0^\circ + i\sin 0^\circ)$$

$$\sqrt[6]{64}\left[\cos\frac{0^\circ + K\cdot 360^\circ}{6} + i\sin\frac{0^\circ + K\cdot 360^\circ}{6}\right]$$

$$= 2\left[\cos K\cdot 60^\circ + i\sin K\cdot 60^\circ\right]$$

$K=0$ $2\,\text{Cis } 0^\circ$

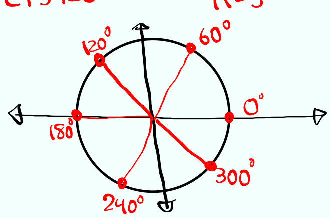
$K=1$ $2\,\text{Cis } 60^\circ$

$K=2$ $2\,\text{Cis } 120^\circ$

$K=3$ $2\,\text{Cis } 180^\circ$

$K=4$ $2\,\text{Cis } 240^\circ$

$K=5$ $2\,\text{Cis } 300^\circ$



Dec 5-11:21 AM